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A COMPARISON OF DEEP NEURAL NETWORK ARCHITECTURES FOR ARRHYTHMIA CLASSIFICATION

Abstract. *In this paper, we describe our studies that compare different types of deep neural networks for automated arrhythmia type classification using the ECG signal. As a source of data and the evaluation benchmark, we took the Physionet Challenge 2017 dataset. In addition to conventional accuracy metrics of the models, we also evaluated the complexities of the models by the number of trainable parameters, to find an optimal trade-off. As a result, our studies bring an evaluation of contemporary deep neural networks architectures on Atrial fibrillation detection task and propose DenseNet as the most efficient in terms of accuracy and computational efficiency.*

Key words: Arrhythmia, AFib, ECG, Neural Network, CNN

Introduction. Nowadays, the classification of ECG signal for heart disease detection is a well-known problem. However, various new methods arise, so it is still a broad field for research. A growth of attention to this problem significantly increased after the publication of Stanford ML group [1], in which they achieved human-level arrhythmia detection using deep neural network (NN) with residual connections. The novelty of the described method was in applying a state-of-the-art NN topology from computer vision to a significant amount of ECG records with corresponding labeled diagnoses. Another advantage of the proposed approach is that it does not require any feature selection as in conventional machine learning (ML) and takes just a raw signal segment as an input. Such an approach is also called “end-to-end machine learning” [2]. Taking into account the success of the mentioned publication, the research community focused on applying this powerful tool using the data from publicly available sources, for instance, PhysioNet web-site [3]. It contains several databases like MIT-BIH Arrhythmia database [4], MIT-BIH Atrial Fibrillation database [3] and PhysioNet Challenge 2017 [5] which include ECG records with atrial fibrillation and can be used for training and testing of machine learning models. This paper will be structured as follows: firstly, in the next chapter, we will describe publications related to arrhythmia classification via different neural networks. Secondly, we will make an overview of existing

publicly-available arrhythmia datasets describing their advantages and drawbacks. Thirdly, in the fourth chapter, we will provide approaches for data preprocessing for neural networks training and in the fifth chapter will describe their characteristics. Next, in the sixth chapter we present a training configuration that was finally applied. In the subsequent chapter we will provide results obtained in several experiments. Finally, in the eighth chapter we will introduce our conclusions based on conducted experiments considering usage of different neural network topologies for arrhythmia detection.

Purpose of the study. There are quite a lot of studies in the field of automated arrhythmia detection; the vast majority of them are using PhysioNet Challenge 2017 dataset and MIT-BIH arrhythmia database as benchmarks. Given papers may be divided into two major groups differing by their data processing pipeline. First type of pipeline is called a “step-by-step” approach and described in [6-10]. It implies whole ECG preprocessing (including annotation) and “manual” feature extraction (mean, std, median, kurtosis, skew, HRV features, wavelet features, features based on lengths of the PQ, ST segments etc.) before classification. The second one is “end-to-end” approach; it implies that the model receives an input signal and produces a prediction without any intermediate steps. The most significant research in this field is previously mentioned in Stanford ML group publication [1]. It is based on a large non-public dataset collected

using iRhythm ECG patch. However, there are also plenty of papers that describe applications of convolutional neural networks on other datasets [11-14], and publications produced by participants of the PhysioNet Challenge [15-18] which showed near the state of the art performance.

Despite the importance of a topic of atrial fibrillation and increased number of discussions after Stanford publication and a new Apple Watch feature release, there is still no unified benchmark for cardiological models (particularly for arrhythmia detection). For instance, the computer vision field has a well-known ImageNet dataset [19], provided by LSVRC challenge, which is used for testing the performance of multiclass image classification. However, there are three most-used datasets (MIT- BIH Arrhythmia, MIT-BIH AF, PhysioNet challenge2017) from PhysioNet platform that is mostly mentioned in papers dedicated to AF detection tasks. Mentioned databases differ from each other by the number of classes, amount of records and ECG leads they were collected from. Detailed Description with empathized pros and cons of each dataset will be provided further in the chapter.

Materials and methods. The MIT-BIH Arrhythmia Database was the first publicly available standardized database that can be used for cardiac dynamics research and performance evaluation of automatic arrhythmia detectors. It was collected during 1975-1980 at Boston's Beth Israel Hospital in collaboration with MIT on 25 male subjects aged 32 to 89 years and 22 female subjects aged 23 to 89 years. A database contains 48 half-hour records recorded from two chest leads (modified limb lead II and modified lead V1). For analysis, preferably first of two recorded leads, since QRS complexes on it have a normal shape. A big advantage of the given dataset is that it has an annotation of each beat (normal beats(N), premature ventricular contractions(V), supraventricular premature beats (S), or a fusion of ventricular and normal beats (F)). At the same time, ECG intervals have annotations of the corresponding rhythm type (Atrial bigeminy, Atrial fibrillation, Atrial flutter, Ventricular bigeminy, 2 heart block, Idioventricular rhythm, Normal Sinus rhythm, Nodal (A-V junctional) rhythm, Paced rhythm, Pre-excitation (WPW), Sinus bradycardia,

Supraventricular tachyarrhythmia, Ventricular trigeminy, Ventricular trigeminy, Ventricular tachycardia). However, like in most medical datasets, both classes of beat types and rhythm types are very unbalanced and underrepresented; this means that records are hardly suitable for training arrhythmia detectors.

MIT-BIH Atrial Fibrillation Database (AFDB) is also one of the datasets collected by Boston's Beth Israel Hospital and MIT. It is quite similar to MIT-BIH AFDB since ECG records were made from the same chest leads (lead II, lead V1). However, there are only 25 ten-hour records with rhythm annotation, whereas 23 of them also contain raw signal. As its title suggests, database records contain atrial fibrillation episodes, in addition, atrial flutter, AV junctional rhythm and normal sinus rhythm are also available. Although a database has a limited number of records, they are quite long and keeping in mind a principle of patient-independent testing, can be sampled to train machine learning models for arrhythmia detection.

PhysioNet/Computing in Cardiology (CinC) challenge 2017 goal was to perform the best classification performance of ECGs recorded on portable single-channel AliveCorec© devices. The entire dataset contains 12,186 labelled training records and 300 validation records (labels were hidden during the challenge and used for leaderboard evaluation). All mentioned records are 9-61 seconds long annotated in one out of four labels (normal sinus rhythm, atrial fibrillation, alternative rhythm, and too noisy to consider it an ECG). Despite the fact that this dataset provides a quite big amount of labelled ECG records, there are a few disadvantages to keep in mind while working with it. First, devices that were used for recording do not require specific electrode placement and accept both LA-RA and RA-LA for the lead I as shown on Image 1. So, about half of the records are inverted in terms of voltage and have to be normalized for further processing. Second, initial dataset labelling had some inaccuracies, since it was made by a single practitioner. This issue has not gone undetected during the challenge, so in the latest update of the dataset, the most untrusted records were revised and relabeled. After evaluating the pros and cons of described datasets, we chose PhysioNet

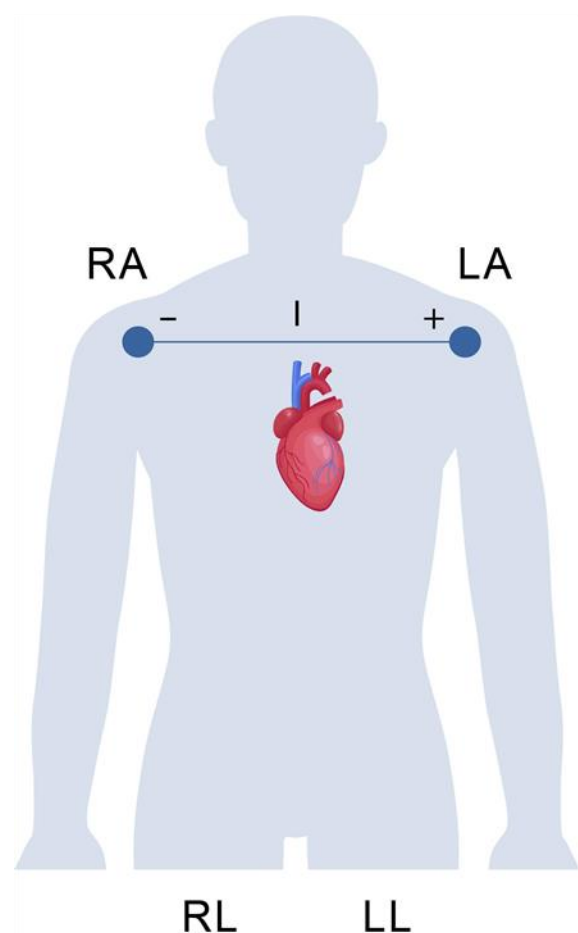
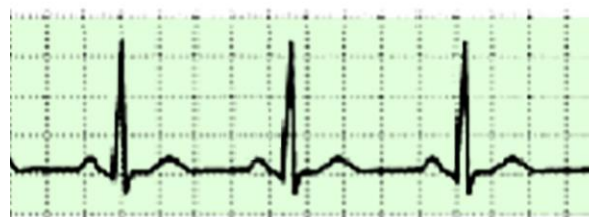


Fig. 1. An electrode placement schema for I ECG lead recording

challenge 2017 dataset for further research. For initial data preprocessing, we used a FIR filter [20] with a cutoff range from 3 Hz to 50 Hz to remove power line and baseline noise. Also, the amplitude of all samples was scaled in the range [0, 1]. As mentioned in chapter 3, the given dataset has an issue with the inverted signal, because of a specific type of ECG recorder. To eliminate this problem and fetch all provided samples, we applied a beat template matching method described by Goodelov [21]. Template matching approach requires signal annotation; thus, Pan-Tompkins [22] algorithm was used. As far as the “noisy” class was underrepresented with only 46 records, it was removed from the analysis. We also experimented with different lengths of dataset samples by taking 15, 30, 45 and 60 seconds of the initial record. If the initial record was long enough to contain several fragments of a certain length, we used them to extend the dataset by sampling multiple samples without overlap from the single record. We will call such dataset configuration as extended. In another

Normal Sinus Rhythm



Atrial Fibrillation



Fig. 2: Normal Sinus Rhythm in comparison with Atrial Fibrillation

case, we just took the first required seconds and cut off the remaining part. Such a dataset we will call simple. For the records smaller than required length symmetric zero-padding was applied.

Rhythm abnormalities are visually well recognizable on the ECG signal as we can see on Image 2. In particular, atrial fibrillation is characterized by absence of P-wave or appearance of the specific f-wave on the QT interval on the standard lead I. Therefore, as was proved before in [23], state of the art approaches from the computer vision, that are based on convolutional neural networks (CNNs), also can be used for mentioned task exploiting 1D convolution operation and show competitive results when compared to conventional feature-based methods. In our research, we decided to test all neural network-based approaches starting from simple multilayer perceptron (MLP) to complex topologies like ResNet, DenseNet and combination of CNN and LSTM. Global Average Pooling was used in CNN models before a softmax layer to reduce dimensionality and have an ability to apply CAM approach [24] for construction of attention maps.

To have a baseline, we decided to take MLP as a predecessor of contemporary neural network-based algorithms. It's known that this method has restrictions related to dramatic increase of computational complexity for big input samples. Taking into account the size of the training set

(approximately 8000 records), a sampling rate of records which is 300 Hz, and length up to 60 seconds, the number of hyperparameters to tune will be too high. To tackle this problem we decided to apply some tricks like downsampling or splitting long ECG fragments to shorter ones, which provided more training data and decreased the complexity of the model. A model itself composed of five layers with 450, 300, 100, 64 (fully-connected), 3 (softmax) nodes, ReLU activation functions [25], and Dropouts [26] with rate 0.5. The amount of nodes for each layer and remaining hyperparameters values were selected based on cross validation.

Convolutional Neural Network is a family of neural networks that uses learned filters to match certain templates in the input data. In terms of image processing, the mentioned square-shaped filter moves vertically and horizontally with certain steps. However, in case of ECG, the data is one-dimensional, accordingly, mentioned filters will also have a single dimension and move only along a time axis. For our studies, we decided to try a simple model with five layers which in turn comprised of 1D convolution, Batch Normalization [27], ReLU as activation, max pooling and dropout with rate 0.5. Kernel sizes varied starting from 12 on the initial layers to 6 on the last one, at the same time, stride were selected in range from 8 to 4.

The reason we decided to apply such architecture is to test the performance of long-short-term memory (LSTM) networks [28], which are highly effective in perceiving temporal dependencies in the data. However, LSTM units are quite computationally expensive, taking into account the size of the ECG segments (i.e. 30 seconds x 500 Hz = 15000 input points). Adding several convolutional layers will solve this issue by reducing the dimensionality and additionally will provide better data representation for LSTM layers. To test this hypothesis we took previously described simple CNN and added a LSTM layer with 100 nodes and a fully connected layer with 65 nodes on top of it before a softmax layer. Given hyperparameters (amount of nodes in the Dense and LSTM layers) were chosen to reach the optimal performance of the model.

ResNet is a specific architecture of deep convolutional neural networks that learns the

residual representation functions instead of direct representation. It represents shortcut (skip) connections that fit an input from the previous layer to the next layer with no changes of the input. Stacking these blocks allows building deeper networks without the occurrence of the vanishing/exploding gradients problems. Initially the given approach showed its advantages on image classification tasks by winning ILSVRC 2015 and MS COCO 2015 in both detection and segmentation. Later, Stanford ML group introduced a study [1] where they trained 34 layer CNN with skip connections on a large dataset of 64,121 ECG records from 29,163 patients. Obtained model outperformed average cardiologist in the classification of 14 different types of heart rhythms disorders. Given paper points to the conclusion that deep CNNs with residual connections are promising techniques for ECG signal classification with different morphology.

DenseNet (densely connected convolutional network) [29] is a topology that shows the state of the art performance in image classification tasks (CIFAR, SVHN, ImageNet) using fewer parameters compared to its closest contender ResNet. Given CNN architecture composed of densely connected blocks (dense blocks). In these blocks, each subsequent layer receives all output feature maps of previous layers, so the number of trainable parameters compared to conventional convolutional models. Such structure gives direct access to the gradient from the loss, which in its turn prevents gradient vanishing/explosion. A major feature of Dense block is that it has the following structure:

- Batch Normalization
- ReLU activation
- Convolution

In which pre-activation happens since batch normalization and activation precede a convolution.

Instead of summing residuals as in ResNet, DenseNet concatenates output feature maps. With no doubts, it's inefficient to concatenate tensors with different sizes. Therefore, all dense blocks have the same feature map size. However, dimensionality compression is essential for CNNs; for these purposes, transition layers are used. They contain Batch Normalization, 1x1

convolution and average pooling. There are suspicious regarding the complexity of the model because concatenation of feature maps from all layers might lead to an extremely high number of parameters. Fortunately, DenseNet has quite narrow layers with up to 32 feature maps, as recommended by authors.

All NN models were implemented on Keras framework [30] (with the TensorFlow backend [31]). Each model was trained during 80 epochs using Adam optimizer with adjustable learning rate and cross-entropy as an optimization loss function. The initial dataset was divided on training, validation and testing split in proportion 70% / 15% / 15% accordingly. To prevent subject overfitting, samples with required length were fragmented before the train/test split.

Results. Given dataset is quite specific and has its limitations, like severe class imbalance. For this reason, we decided to experiment not only with neural networks types and configurations but also with the formation of classes merging or not using them in particular training experiments. Conducting described experiments, we observed interesting results that will be described further in this section. According to 2017 PhysioNet Challenge, average F1-score was used to evaluate the performance of participants, so we decided to follow the same rule for evaluation of our models. Firstly, we trained MLP to have a baseline for our further experiments. To train MLP, we had to overcome a curse of dimensionality, so the sampling rate was reduced from 300 Hz to 30 Hz Fourier method [32]. As expected, MLP tended to overfit and showed the worst performance compared to other models. Next, we decided to build a simple four-layer CNN with filters of bigger size and stride to provide a sufficient down-sampling before global average pooling and softmax activation. A global average pooling was used in most experiments to reduce a total number of parameters in the model and to have an ability of model interpretation using CAM (class activation map) approach [24]. As shown on the table, the simple model performed better than MLP (baseline), but still with low accuracy. Next, we attached a LSTM layer with 64 blocks between the simple model that was described earlier and a fully connected layer with 64 neurons that precedes a softmax activation. This configuration

performed slightly better than the previous model, but still poor classification accuracy. Our further steps were to try state-of-the-art (SOTA) topologies like ResNet and DenseNet to solve the given classification task. First, we decided to train ResNet as an earlier version CNN with indirect connections. A model contained five subsequently connected residual blocks with global average pooling on top before the softmax layer. Such topology performed much better and showed a result that is quite close to the top of the leaderboard in the PhysioNet/Computing in Cardiology Challenge 2017. Finally, we trained a DenseNet which contained five dense blocks and growth rate 12. It showed the best results with F1-score 0.834 and accuracy. All results for both simple (S) and extended (E) test datasets are shown in Table 1.

Besides classification performance, model complexity is also an important characteristic to analyze. Since we experimented with different types of NN, correspondingly, their complexities also varied. The number of parameters for each model is shown in Table 2. As we can see, the simplest model, in terms of the number of trained parameters, is DenseNet, and the most complex model is ResNet. Taking into account both accuracy and complexity metrics, we selected a DenseNet configuration as far as it is both the most accurate and the most light-weight model for solving given classification problems.

Conclusion. In this work, we trained various deep neural network models starting from simple multilayer perceptron to SOTA topologies like ResNet and DenseNet to classify ECG fragments on Normal Sinus rhythm, AFib rhythm and Other rhythms on different dataset configurations. The best test results were obtained on DenseNet using simple 45-second signal fragments. It is worth mentioning that ResNet showed relatively close performance. However, DenseNet has one significant advantage, thanks to its structural characteristics, it has significantly fewer parameters, so can be considered as more computationally efficient. Furthermore, the high efficiency of such algorithms will allow their integration directly into the stationary cardiographs and even into the wearable devices, which would bring the diagnostics of arrhythmias on the next level. Moreover, this work covers

Table 1.

F1-score of trained NN models

Window size								
Preprocessing	15		30		45		60	
	S	E	S	E	S	E	S	E
MLP	0.2555	0.2590	0.2520	0.2492	0.2520	0.2487	0.2520	0.2539
CNN	0.5574	0.6202	0.6095	0.6414	0.6138	0.6456	0.5963	0.5991
CNN+LSTM	0.5636	0.6477	0.6244	0.6649	0.6491	0.6511	0.6508	0.6349
ResNet	0.7314	0.7905	0.8090	0.8125	0.8252	0.8254	0.8226	0.81168
DenseNet	0.7491	0.8008	0.8206	0.8111	0.8341	0.8163	0.8221	0.8221

Table 2.

Number of trainable parameters in different NNmodels

	Window size			
	15	30	45	60
MLP	379509	575709	782709	978909
CNN	315907	315907	315907	315907
CNN+ LSTM	353091	353091	353091	353091
ResNet	1065347	1065347	1065347	1065347
DenseNet	186364	186364	186364	186364

most of the novel deep learning architectures and defines a new baseline starting point for further research in Atrial fibrillation detection based on the ECG signal.

References

1. Awni Hannun, Pranav Rajpurkar, Masoumeh Haghpanahi, Geoffrey H. Ti-son, Codie Bourn, Mintu Turakhia, and Andrew Y. Ng. *Cardiologist-level arrhythmia detection and classification in ambulatory electrocardiograms using a deep neural network.* *Nature Medicine*, 25, 01 2019.
2. Ilya Sutskever, Oriol Vinyals, and Quoc V Le. *Sequence to sequence learning with neural networks.* In Z. Ghahramani, M. Welling, C. Cortes, N. D. Lawrence, and K. Q. Weinberger, editors, *Advances in Neural Information Processing Systems 27*, pages 3104–3112. Curran Associates, Inc., 2014.
3. A. L. Goldberger, L. A. N. Amaral, L. Glass, J.

4. M. Hausdorff, P. Ch.Ivanov, R. G. Mark, J. E. Mietus, G. B. Moody, C.-K. Peng, and H. E. Stanley. *PhysioBank, PhysioToolkit, and PhysioNet: Components of a new research resource for complex physiologic signals.* *Circulation*, 101(23):e215–e220, 2000 (June 13). *Circulation Electronic Pages*: <http://circ.ahajournals.org/content/101/23/e215.full> PMID:1085218; doi:10.1161/01.CIR.101.23.e215.
4. G. B. Moody and R. D. Davies Mark. *The impact of the mit-bih arrhythmia database.* *IEEE Engineering in Medicine and Biology Magazine*, 20:45–50, 2001.
5. Gari D. Clifford, Changchun Liu, Benjamin Moody, Li wei H. Lehman, Ikaro Silva, Qiao Li, Alistair Edward William Johnson, and Roger G. Mark. *Af classification from a short single lead ecg recording: The Physionet/computing in*

cardiology challenge 2017.2017 Computing in Cardiology (CinC), pages 1–4, 2017.

6. F. Alonso-Atienza, E. Morgado, L. Fernández-Martínez, A. García-Alberola, and J. L. Rojo-Álvarez. Detection of life-threatening arrhythmias using feature selection and support vector machines. *IEEE Transactions on Biomedical Engineering*, 61(3):832–840, March 2014.

7. Kemal Polat and Salih Güneş. Detection of ecg arrhythmia using a differential expert system approach based on principal component analysis and least square support vector machine. *Applied Mathematics and Computation*, 186(1):898 – 906, 2007.

8. Juan Ródenas García, Manuel Garcia, Raúl Alcaraz, and Jose Rieta. Wavelet entropy automatically detects episodes of atrial fibrillation from single-lead electrocardiograms. *Entropy*, 17:6179–6199, 09 2015.

9. Mjaye Mazwi, Sebastian Goodfellow, Andrew Goodwin, Robert Greer, Peter C. Laussen, and Danny Eytan. Atrial fibrillation classification using step-by-step machine learning. *Biomedical Physics Engineering Express*, 4, 04 2018.

10. Deeptankar Demazumder, Douglas Lake, Alan Cheng, Travis Moss, Eliseo Guallar, Robert Weiss, Steven Jones, Gordon F Tomaselli, and Joseph Moorman. Dynamic analysis of cardiac rhythms for discriminating atrial fibrillation from lethal ventricular arrhythmias. *Circulation. Arrhythmia and electrophysiology*, 6, 05 2013.

11. U. Rajendra Acharya, Hamido Fujita, Oh Shu Lih, Yuki Hagiwara, Jen Hong Tan, and Muhammad Adam. Automated detection of arrhythmias using different intervals of tachycardia ecg segments with convolutional neural network. *Information Sciences*, 405:81 – 90, 2017.

12. U. Rajendra Acharya, Hamido Fujita, Oh Shu Lih, Yuki Hagiwara, Jen Hong Tan, and Muhammad Adam. Automated detection of arrhythmias using different intervals of tachycardia ecg segments with convolutional neural network. *Information Sciences*, 405:81 – 90, 2017.

13. B. Pourbabae, M. J. Roshtkhari, and K. Khorasani. Feature leaning with deep convolutional neural networks for screening patients with paroxysmal atrial fibrillation. In 2016

International Joint Conference on Neural Networks (IJCNN), pages 5057–5064, July 2016.

14. Rishikesan Kamaleswaran, Ruhi Mahajan, and Oguz Akbilgic. A robust deep convolutional neural network for the classification of abnormal cardiac rhythm using varying length single lead electrocardiogram. *Physiological Measurement*, 39, 01 2018.

15. Rishikesan Kamaleswaran, Ruhi Mahajan, and oguz akbilgic. A robust deep convolutional neural network for the classification of abnormal cardiac rhythm using varying length single lead electrocardiogram. *Physiological Measurement*, 39, 01 2018.

16. Martin Zihlmann, Dmytro Perekrestenko, and Michael Tschannen. Convolutional recurrent neural networks for electrocardiogram classification. 10 2017.

17. Jonathan Rubin, Saman Parvaneh, Asif Rahman, Bryan Conroy, and Saeed Babaeizadeh. Densely connected convolutional networks for detection of atrial fibrillation from short single-lead ecg recordings. *Journal of Electrocardiology*, 51, 08 2018.

18. Philip Warrick and Masun Nabhan Homs. Cardiac arrhythmia detection from ecg combining convolutional and long short-term memory networks. 09 2017.

19. J. Deng, W. Dong, R. Socher, L.-J. Li, K. Li, and L. Fei-Fei. ImageNet: A Large-Scale Hierarchical Image Database. In *CVPR09*, 2009.

20. K. S. Kumar, B. Yazdanpanah, and P. R. Kumar. Removal of noise from electrocardiogram using digital fir and iir filters with various methods. In 2015 International Conference on Communications and Signal Processing (ICCSP), pages 0157–0162, April 2015.

21. Sebastian Goodfellow, Andrew Goodwin, Danny Eytan, Robert Greer, Mjaye Mazwi, and Peter Laussen. Towards understanding ecg rhythms classification using convolutional neural networks and attention mappings. 08 2018.

22. J. Pan and W. J. Tompkins. A real-time qrs detection algorithm. *IEEE Transactions on Biomedical Engineering*, BME-32(3):230–236, March 1985.

23. Alex Krizhevsky, Ilya Sutskever, and Geoffrey E. Hinton. Imagenet classification with deep convolutional neural networks. *Neural Information Processing Systems*, 25, 01 2012.

24. Bolei Zhou, Aditya Khosla, Àgata Lapedriza, Aude Oliva, and AntonioTorralba. Learning deep features for discriminative localization. 12 2015.
25. Xavier Glorot, Antoine Bordes, and Yoshua Bengio. Deep sparse rectifier neural networks. In Geoffrey Gordon, David Dunson, and Miroslav Dudík, editors, *Proceedings of the Fourteenth International Conference on Artificial Intelligence and Statistics*, volume 15 of *Proceedings of Machine Learning Research*, pages 315–323, Fort Lauderdale, FL, USA, 11–13 Apr 2011. PMLR.
26. Nitish Srivastava, Geoffrey E. Hinton, Alex Krizhevsky, Ilya Sutskever, and Ruslan R. Salakhutdinov. Dropout: a simple way to prevent neural networks from overfitting. *Journal of Machine Learning Research*, 15:1929–1958, 2014.
27. Sergey Ioffe and Christian Szegedy. Batch normalization: Accelerating Deep network training by reducing internal covariate shift. 02 2015.
28. F. A. Gers, J. Schmidhuber, and F. Cummins. Learning to forget: continual prediction with lstm. In 1999 Ninth International Conference on Artificial Neural Networks ICANN 99. (Conf. Publ. No. 470), volume 2, pages 850–855 vol.2, Sep. 1999.
29. Gao Huang, Zhuang Liu, Laurens van der Maaten, and Kilian Q. Weinberger. Densely connected convolutional networks. In *The IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, July 2017.
30. François Chollet et al. Keras. <https://github.com/fchollet/keras>, 2015.
31. Martín Abadi, Ashish Agarwal, Paul Barham, Eugene Brevdo, Zhifeng Chen, Craig Citro, Greg S. Corrado, Andy Davis, Jeffrey Dean, Matthieu Devin, Sanjay Ghemawat, Ian Goodfellow, Andrew Harp, Geoffrey Irving, Michael Isard, Yangqing Jia, Rafal Jozefowicz, Lukasz Kaiser, Manjunath Kudlur, Josh Levenberg, Dan Mané, Rajat Monga, Sherry Moore, Derek Murray, Chris Olah, Mike Schuster, Jonathon Shlens, Benoit Steiner, Ilya Sutskever, Kunal Talwar, Paul Tucker, Vincent Vanhoucke, Vijay Vasudevan, Fernanda Viégas, Oriol Vinyals, Pete Warden, Martin Wattenberg, Martin Wicke, Yuan Yu, and Xiaoqiang Zheng. *TensorFlow: Large-scale machine learning on heterogeneous systems*, 2015. Software available from tensorflow.org.
32. W. G. Hawkins. Fourier transform resampling: theory and application [medical imaging]. In 1996 IEEE Nuclear Science Symposium. Conference Record, volume 3, pages 1491–1495 vol.3, Nov 1996